# NS Matter Sliver

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**Abstract**

Using algebraic methods the Neveu-Schwarz fermionic matter sliver is constructed. Inspirited by the wedge algebra both equations for the sliver, linear and quadratic, are considered. It is shown that both equations give the same nontrivial answer. The sliver is considered also using CFT methods where it is defined as the limit of the wedge states in the NS sector of the superstring.

# Contents



# 1 Introduction

The cubic open string field theory around the tachyon vacuum, the vacuum string field theory (VSFT), has been proposed in [1] and is investigated now very intensively [2]-[27]. VSFT action has the same form as the original Witten SFT action [28]

$$S[\Phi] = \frac{1}{g_o^2}\left[\frac{1}{2}\int \Phi \star Q\Phi + \frac{1}{3}\int \Phi \star \Phi \star \Phi\right], \tag{1.1}$$

but with a new differential operator $\mathcal{Q}$ (for review of SFT see [29, 30, 31]). VSFT is obtained by a shift of the original Witten SFT action to the tachyon vacuum. The absence of physical open string excitations around the tachyon vacuum [32], [33, 34, 35, 36] motivates a suggestion [1] that after some field redefinition $\mathcal{Q}$ can be written as a pure ghost operator. If this conjecture is true it is worth to search for the solutions of VSFT equation of motion

$$\mathcal{Q}\Phi + \Phi \star \Phi = 0 \tag{1.2}$$

in the factorized form $\Phi = \Xi_{matter} \otimes \Phi_{ghost}$, where $\Xi_{matter}$ satisfies a projector-like equation:

$$\Xi_{matter} = \Xi_{matter} \star\ \Xi_{matter}. \tag{1.3}$$

An equation similar to (1.3) has appeared in a construction of solitonic solutions in noncommutative field theories in the large non-commutativity limit [37].

A way to solve projection equation (1.3) has been proposed by Rastelli and Zwiebach [2]. They have constructed a solution to (1.3) as the $n \to \infty$ limit of the wedge states $|n\rangle$. The wedge states are defined on CFT language and they satisfy the algebra

$$|n\rangle \star |m\rangle = |n+m-1\rangle. \tag{1.4}$$

From algebra (1.4) it immediately follows that $|\infty\rangle$, the so-called sliver state, satisfies (1.3).

A solution of (1.3) has been also constructed in the oscillator formalism by Kostelecky and Potting [3]. The state $|\infty\rangle$ has been identified with the solution [3] numerically [4] and later on by direct calculations [13]. The sliver state and its generalizations are studied in [2]-[27]. The sliver state is a special one in a sense that it can be defined in an arbitrary boundary CFT [2, 8]. The projector-like form of eq.(1.3) gives a new impulse for the development of the half-string formalism [5, 6, 7], which drastically simplifies Witten's $\star$-product (see review [31]). The previous consideration deals only with the bosonic SFT.

The subject of the present paper is a search for the sliver state for the fermionic string. Open fermionic string in the NSR formalizm has a tachyon in GSO$-$ sector that leads to a classical instability of the perturbative vacuum in the theory without supersymmetry. It has been proposed [32] to interpret the tachyon condensation in GSO$-$ sector of the NS string as a decay of unstable non-BPS D-brane. By using the level truncation scheme it was shown that the tachyon potential in the open cubic NS SFT has a nontrivial minimum. In the framework of the Sen interpretation the vacuum energy of the open NS GSO$-$ string cancels the tension of the unstable space-filling 9-brane. This cancellation has been checked [38].

The cubic action for the NS GSO+ string field [39, 40] can be be easily generalized to both GSO+ and GSO$-$ sectors

$$S[\mathcal{A}] = \frac{1}{g_o^2}\left[\frac{1}{2}\int' \mathcal{A}\star Q\mathcal{A} + \frac{1}{3}\int' \mathcal{A}\star\mathcal{A}\star\mathcal{A}\right], \tag{1.5}$$

where $g_o$ is a coupling constant and the NS string integral is modified accounting the "measure" $Y_{-2}$: $\int' = \int Y_{-2}$. Here $Y_{-2}$ is a double-step inverse picture changing operator. $\mathcal{A}$ describes both GSO+ and GSO$-$ fields. As in the bosonic case one can assume that VSSFT (vacuum super string filed theory) is described by the cubic action (1.5) with a pure ghost $\mathcal{Q}$. Hence, there is a reason to search for solutions of the VSSFT equation of motion

$$\mathcal{Q}\mathcal{A} + \mathcal{A}\star\mathcal{A} = 0 \tag{1.6}$$

in the factorized form

$$\mathcal{A} = \Xi^X \otimes \Xi^\psi \otimes \mathcal{A}_{ghost}. \tag{1.7}$$

Note that the VSSFT equation of motion in the -1 picture [28] would have the form

$$\mathcal{Q}\mathcal{A} + \mathrm{X}\mathcal{A}\star\mathcal{A} = 0. \tag{1.8}$$

Because of the insertion of the picture-changing operator $X$ mixing ghost and matter fields solutions of (1.8) do not admit the factorized form.

The purpose of the present paper is a construction of the solution of fermionic projector-like equation

$$|\Xi^{\psi}\rangle \star |\Xi^{\psi}\rangle = |\Xi^{\psi}\rangle. \tag{1.9}$$

Inspirited by the bosonic wedge algebra (1.4) we algebraically construct a solution of the following equation

$$|\Xi^{\psi}\rangle \star |0\rangle = |\Xi^{\psi}\rangle, \tag{1.10}$$

where $|0\rangle$ is a vacuum in the fermionic sector. We show that (1.10) and (1.9) have the same nontrivial solutions.

The NS matter sliver is constructed also using CFT methods. Numerical calculations give the remarkable evidence that two definitions of the sliver, conformal and algebraic, are identical.

# 2 Preliminary

Fermionic coordinates in the Neveu-Schwarz sector are defined for $\sigma \in [0, \pi]$ and are given by

$$\psi^{\mu}_{\pm}(\sigma) = \sum_{r=n+\frac{1}{2}} e^{\pm\imath r\sigma}\psi^{\mu}_{r}. \tag{2.1}$$

They satisfy the following anticommutation relations

$$\{\psi^{\mu}_{r}, \psi^{\nu}_{s}\} = \delta_{r+s,0}g^{\mu\nu}. \tag{2.2}$$

The fermionic squeezed states have the form

$$|S\rangle = \exp(\frac{1}{2}\psi^{\dagger}_{r}S_{rs}\psi^{\dagger}_{s})|0\rangle, \tag{2.3}$$

One has

$$\begin{aligned}\langle 0|\exp(\frac{1}{2}\psi_{r}S_{sr}\psi_{s})\exp(\psi^{\dagger}_{r}\mu_{r} + \frac{1}{2}\psi^{\dagger}_{r}V_{rs}\psi^{\dagger}_{s})|0\rangle\\ = \det(1 + S_{rl}V_{ls})^{1/2}\exp(\frac{1}{2}\mu_{r}(1 + S_{rl}V_{lk})^{-1}S_{ks}\mu_{s}),\end{aligned} \tag{2.4}$$

where $\mu_r$ are the anticommuting parameters. The state $\langle A|$ is the BPZ conjugate of the state $|A\rangle$ and is given by

$$_{1}\langle A| = \;_{12}\langle R|A\rangle_{2}, \tag{2.5}$$

where ${}_{12}\langle R|$ is the reflector [42]

$$ {}_{12}\langle R| = {}_{12}\langle 0| \exp(-\imath \psi^1_r (-)^r \psi^2_r). \tag{2.6} $$

The BPZ conjugate of (2.3) is

$$ \langle S| = \langle 0| \exp(-\frac{1}{2}\psi_r C_{rl} S_{lk} C_{ks} \psi_s), \tag{2.7} $$

where $C_{rs} = (-1)^r \delta_{rs}$ and $C^2 = -1$.

Following Gross and Jevicki [41, 42] we represent the star product of the states $|A\rangle$ and $|B\rangle$ in the Neveu-Schwarz sector of the matter fermionic SFT as

$$ |A\rangle \star |B\rangle_1 = {}_2\langle A|\, {}_3\langle B|V_3\rangle, \tag{2.8} $$

where the three string vertex is given by

$$ |V_3\rangle = \exp(\frac{1}{2}\psi^{a\,\dagger}_r V^{ab}_{rs} \psi^{b\,\dagger}_s)|0\rangle_{123}, \tag{2.9} $$

where

$$ V^{a\,a} = \frac{1}{3}(I + U + CUC), \tag{2.10a} $$
$$ V^{a\,a+1} = \frac{1}{3}(I + \alpha U + \alpha^* CUC), \tag{2.10b} $$
$$ V^{a\,a-1} = \frac{1}{3}(I + \alpha^* U + \alpha CUC), \tag{2.10c} $$

and $\alpha = e^{2\pi\imath/3}$. The three string vertex has the cyclic property $V^{ab} = V^{(a+1)(b+1)}$, where indexes $a, b$ are defined mod(3). Matrices $I$ and $U$ are calculated in appendix A and are given by

$$ I = \frac{\tilde{F}}{1-F}, \qquad U = -\frac{\tilde{F} - \sqrt{3}C}{2+F}, \qquad CUC = -\frac{\tilde{F} + \sqrt{3}C}{2+F}. \tag{2.11} $$

Here $F$ and $\tilde{F}$ are hermitian matrices [42]

$$ F_{rs} = -\frac{2}{\pi}\frac{\imath^{r-s}}{r+s}, \qquad r = s \bmod(2), \tag{2.12} $$
$$ \tilde{F}_{rs} = \frac{2}{\pi}\frac{\imath^{r+s}}{s-r}, \qquad r = s+1 \bmod(2) \tag{2.13} $$

with the following properties

$$ F^2 - \tilde{F}^2 = 1, \quad [F, \tilde{F}] = 0, \tag{2.14} $$
$$ CFC = -F, \quad F^T = F, \quad C\tilde{F}C = \tilde{F}, \quad \tilde{F}^T = -\tilde{F}. \tag{2.15} $$

In terms of $F$, $\tilde{F}$ and $C$ the three string vertex is given by

$$V^{11} = \frac{F\tilde{F}}{(1-F)(2+F)}, \tag{2.16a}$$

$$V^{12} = \frac{\tilde{F} + \imath C(1-F)}{(1-F)(2+F)}, \tag{2.16b}$$

$$V^{21} = \frac{\tilde{F} - \imath C(1-F)}{(1-F)(2+F)}. \tag{2.16c}$$

We will use the following notation

$$M_{ab} = CV^{ab}. \tag{2.17}$$

One has the following properties for $M_{ab}$

$$M_{12} + M_{21} + M_{11} = CI, \tag{2.18a}$$

$$[M_{ab}, M_{cd}] = 0, \quad \forall \;\; a, b, c, d, \tag{2.18b}$$

$$M_{12}M_{21} = M_{11}^2 - CI^{-1}M_{11}. \tag{2.18c}$$

# 3 Algebraic Construction

## 3.1 Linear equation

Let us assume that $|\Xi^\psi\rangle$ has a squeezed form (2.3)

$$|\Xi^\psi\rangle = \mathcal{N}^{10} \exp(\frac{1}{2}\psi_r^\dagger S_{rs} \psi_s^\dagger)|0\rangle, \tag{3.1}$$

and let us solve the following equation

$$|\Xi^\psi\rangle \star |0\rangle = |\Xi^\psi\rangle. \tag{3.2}$$

Here $\mathcal{N}$ is the normalization factor to be specified later. Using (2.4) one gets the following equation

$$M_{21}(1 - TM_{11})^{-1}TM_{12} + M_{11} = T. \tag{3.3}$$

Here we denote $T = CS$. Assuming the following commutation relations

$$[T, M_{ab}] = 0, \quad \forall \;\; a, b, \tag{3.4}$$

and using the properties (2.18), equation (3.3) can be rewritten as

$$T^2 M_{11} - T(1 + CI^{-1}M_{11}) + M_{11} = 0. \tag{3.5}$$

An explicit solution of this equation is

$$T = \frac{1 + CI^{-1}M_{11} - \sqrt{(1 + CI^{-1}M_{11})^2 - 4M_{11}^2}}{2M_{11}}. \tag{3.6}$$

We choose the minus sign for the square root in (3.6). We will check later this choice of the sign.

## 3.2 Quadratic equation

Let us now consider the projector-like equation

$$|\Xi^{\psi}\rangle \star |\Xi^{\psi}\rangle = |\Xi^{\psi}\rangle, \tag{3.7}$$

where $|\Xi^{\psi}\rangle$ has the form (3.1). Using (2.4) one gets from (3.7) the following equation

$$(M_{12}, M_{21}) \left(1 - T \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\right)^{-1} \begin{pmatrix} TM_{21} \\ TM_{12} \end{pmatrix} + M_{11} = T. \tag{3.8}$$

This equation can be rewritten in the form

$$(T - CI)(T^2 M_{11} - T(1 + CI^{-1} M_{11}) + M_{11}) = 0. \tag{3.9}$$

We get a trivial solution $T = CI$. It corresponds to the identity state $|I\rangle$. The identity state is the identity of the star algebra and has the form [42]

$$|I\rangle = \exp(\frac{1}{2}\psi_r^{\dagger} I_{rs} \psi_s^{\dagger})|0\rangle. \tag{3.10}$$

Up to the factor $(T - CI)$, equation (3.9) is the same as equation (3.3).

Finally, one gets the following expression for the NS matter sliver

$$|\Xi^{\psi}\rangle = \mathcal{N}^{10} \exp(\frac{1}{2}\psi_r^{\dagger} S_{rs} \psi_s^{\dagger})|0\rangle, \tag{3.11}$$

where one can find normalization factor from eq. (3.7)

$$\begin{aligned} \mathcal{N} &= \det((1 - TM_{11})^2 - T^2 M_{12} M_{21})^{-1/2} \\ &= \det(1 - CI^{-1} M_{11} + T(CI^{-1} - 2M_{11} + I^{-2} M_{11}))^{-1/2}. \end{aligned} \tag{3.12}$$

Using the normalization $\langle 0|0\rangle = 1$, one can now write the norm of the NS sliver state

$$\langle \Xi^{\psi}|\Xi^{\psi}\rangle = \mathcal{N}^{20} \det(1 - S^2)^5. \tag{3.13}$$

# 4 NS Wedge States

A generalization of the bosonic wedge states [2, 4, 8] to the fermionic wedge states is straightforward. Wedge states $|n\rangle$ are defined by

$$\langle n|\phi^{\psi}\rangle = \langle f_n \circ \phi^{\psi}(0)\rangle, \tag{4.1}$$

where $|\phi^{\psi}\rangle$ is an arbitrary state which belongs to the fermionic subspace, $f_n \circ \phi^{\psi}(\xi)$ denotes the conformal transform of $\phi^{\psi}(\xi)$ and $f_n(\xi)$ is the same as in the bosonic case, i.e.

$$f_n(\xi) = \frac{n}{2} \tan\left(\frac{2}{n} \tan^{-1} \xi\right). \tag{4.2}$$

The wedge state multiplication rule can be shown to be [2]

$$|n\rangle \star |m\rangle = |n+m-1\rangle. \tag{4.3}$$

The wedge state with $n=1$ corresponds to the identity of the star algebra and with $n=2$ corresponds to the vacuum.

Taking the limit $n \to \infty$ in (4.2) one derives the conformal map for the sliver state $|\infty\rangle$

$$w(\xi) = \tan^{-1}(\xi). \tag{4.4}$$

Following [42, 43, 44] one gets the explicit formula for the state $|\Lambda\rangle$ corresponding to a conformal map $\lambda(\xi)$

$$|\Lambda\rangle \propto \exp(\frac{1}{2}\psi_r^\dagger \Lambda_{rs} \psi_s^\dagger)|0\rangle, \tag{4.5}$$

$$\Lambda_{rs} = \oint \frac{d\xi}{2\pi\imath} \oint \frac{d\xi'}{2\pi\imath} \xi^{-r-\frac{1}{2}} \xi'^{-s-\frac{1}{2}} \left(\frac{\partial\lambda(\xi)}{\partial\xi}\right)^{\frac{1}{2}} \frac{1}{\lambda(\xi)-\lambda(\xi')} \left(\frac{\partial\lambda(\xi')}{\partial\xi'}\right)^{\frac{1}{2}}. \tag{4.6}$$

Here $\oint$ denotes the contour integration around the origin. Using the sliver conformal map (4.4) one gets

$$\left(\frac{\partial w(\xi)}{\partial\xi}\right)^{\frac{1}{2}} \frac{1}{w(\xi)-w(\xi')} \left(\frac{\partial w(\xi')}{\partial\xi'}\right)^{\frac{1}{2}} = \frac{2\imath}{\sqrt{1+\xi^2}\sqrt{1+\xi'^2}} \ln\left(\frac{(1+i\xi)(1-i\xi')}{(1-i\xi)(1+i\xi')}\right). \tag{4.7}$$

So the conformal sliver $|\tilde{\Xi}^\psi\rangle \equiv |\infty\rangle$ is defined as

$$|\tilde{\Xi}^\psi\rangle = \tilde{\mathcal{N}}^{10} \exp(\frac{1}{2}\psi_r^\dagger \tilde{S}_{rs} \psi_s^\dagger)|0\rangle, \tag{4.8}$$

$$\tilde{S}_{rs} = \oint \frac{d\xi}{2\pi\imath} \oint \frac{d\xi'}{2\pi\imath} \xi^{-r-\frac{1}{2}} \xi'^{-s-\frac{1}{2}} \frac{2\imath}{\sqrt{1+\xi^2}\sqrt{1+\xi'^2}} \ln\left(\frac{(1+i\xi)(1-i\xi')}{(1-i\xi)(1+i\xi')}\right). \tag{4.9}$$

The matrix $\tilde{S}_{rs}$ can be calculated explicitly. Only coefficients with $r+s=even$ differ from zero.

## 5 Comparison

Results of calculation for the algebraic sliver $|\Xi^\psi\rangle$ that are presented on Table 1 are in accord with the results of calculations for the conformal sliver $|\tilde{\Xi}^\psi\rangle$ presented below

$$\tilde{S}_{\frac{1}{2}\frac{3}{2}} = \frac{1}{6} \approx 0.16667, \qquad \tilde{S}_{\frac{1}{2}\frac{7}{2}} = -\frac{43}{360} \approx -0.11944, \tag{5.1a}$$

$$\tilde{S}_{\frac{1}{2}\frac{11}{2}} = \frac{1459}{15120} \approx 0.09649, \qquad \tilde{S}_{\frac{5}{2}\frac{3}{2}} = -\frac{1}{40} = -0.02500, \tag{5.1b}$$

$$\tilde{S}_{\frac{5}{2}\frac{7}{2}} = \frac{239}{7560} \approx 0.03161, \qquad \tilde{S}_{\frac{5}{2}\frac{11}{2}} = -\frac{18947}{604800} \approx -0.03133. \tag{5.1c}$$

We see a conspicuous agreement between $S_{rs}$ and $\tilde{S}_{rs}$. This gives a convincing evidence that two descriptions of the NS sliver, algebraic and conformal, are identical. Normalization factors $\mathcal{N}$ and $\tilde{\mathcal{N}}$ should be equal since these two states are normalized identically.

| $L$ | $S_{\frac{1}{2}\frac{3}{2}}$ | $S_{\frac{1}{2}\frac{7}{2}}$ | $S_{\frac{1}{2}\frac{11}{2}}$ | $S_{\frac{5}{2}\frac{3}{2}}$ | $S_{\frac{5}{2}\frac{7}{2}}$ | $S_{\frac{5}{2}\frac{11}{2}}$ |
|---|---|---|---|---|---|---|
| 30 | 0.13744 | −0.09233 | 0.07098 | −0.01261 | 0.01996 | −0.02026 |
| 60 | 0.14667 | −0.09950 | 0.07696 | −0.01658 | 0.02305 | −0.02286 |
| 100 | 0.15306 | −0.10471 | 0.08146 | −0.01940 | 0.02535 | −0.02484 |
| 130 | 0.15616 | −0.10729 | 0.08372 | −0.02078 | 0.02650 | −0.02585 |
| Exact | 0.16667 | −0.11944 | 0.09649 | −0.02500 | 0.03161 | −0.03133 |

Table 1: Numerical results for the elements of the matrix $S_{rs}$. We compute $S_{rs}$ by restricting the indices $r, s$ of $V^{11}_{rs}$ and $I^{-1}_{rs}$ to be less or equal to $L$ so that $V^{11}$ and $I^{-1}$ are $L\times L$ matrices, and then using equation (3.6). In the last line the results (5.1) are presented.

## 6 Conclusion

In this paper we have constructed the fermionic matter part of the VSSFT equations. The VSSFT action (1.5) on a factorized solution of the equations of motion is:

$$S|_\Xi = -\frac{1}{6g_o^2}\langle\Xi^X|\Xi^X\rangle\langle\Xi^\psi|\Xi^\psi\rangle\langle\langle Y_{-2}|\mathcal{A}_{gh}, \mathcal{Q}\mathcal{A}_{gh}\rangle\rangle. \tag{6.1}$$

Eq. (6.1) shows that the matter fermionic part drop out from the ratio of tensions of D-$(9-k)$ and D-$(8-k)$ branes. Indeed, the ratio of the actions associated with the solutions representing D-branes of different dimensions is given by

$$\frac{S|_{\Xi'}}{S|_\Xi} = \frac{\langle\Xi'^X|\Xi'^X\rangle}{\langle\Xi^X|\Xi^X\rangle}, \tag{6.2}$$

since these D-branes have the same matter fermionic and ghost parts. The ghost and matter fermionic parts drop out both for momentum dependent and momentum independent solutions. In [4] it is argued that in the bosonic case the ratio of tensions of D-$(25-k)$ and D-$(24-k)$ branes given by (6.2) is equal to $1/2\pi$. A similar result one gets for the ratio of tensions in the VSSFT

$$\frac{\tau_{9-k}}{\tau_{8-k}}=\frac{1}{2\pi}. \tag{6.3}$$

Assuming that the tachyonic kinks describe lower dimensional D-branes we do expect the relation (6.3).

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# A NS String Overlaps

The three string overlaps [42] are

$$\begin{aligned}\psi^i_+(\sigma)&=\imath\psi^{i-1}_+(\pi-\sigma),\quad 0\le\sigma\le\frac{\pi}{2},\\ \psi^i_-(\sigma)&=-\imath\psi^{i-1}_-(\pi-\sigma),\quad 0\le\sigma\le\frac{\pi}{2}\end{aligned} \tag{A.4}$$

and

$$\begin{aligned}\psi^{i-1}_+(\sigma)&=-\imath\psi^{i}_+(\pi-\sigma),\quad \frac{\pi}{2}\le\sigma\le\pi,\\ \psi^{i-1}_-(\sigma)&=\imath\psi^{i}_-(\pi-\sigma),\quad \frac{\pi}{2}\le\sigma\le\pi.\end{aligned} \tag{A.5}$$

In terms of $\psi^i(\sigma)$ overlaps can be rewritten in the following way

$$-\pi\le\sigma\le\pi,\qquad \psi^i(\sigma)=\begin{cases} i\psi^{i+1}(-\sigma-\pi), & -\pi\le\sigma\le-\frac{\pi}{2},\\ -i\psi^{i-1}(-\sigma-\pi), & -\frac{\pi}{2}\le\sigma\le 0,\\ i\psi^{i-1}(\pi-\sigma), & 0\le\sigma\le\frac{\pi}{2},\\ -i\psi^{i+1}(\pi-\sigma), & \frac{\pi}{2}\le\sigma\le\pi.\end{cases} \tag{A.6}$$

Introducing $Z_3$ Fourier coordinates

$$\Psi^1=\frac{1}{\sqrt{3}}(\psi^1_++\psi^2_++\psi^3_+), \tag{A.7a}$$

$$\Psi^2=\frac{1}{\sqrt{3}}(\psi^1_++\alpha\psi^2_++\alpha^*\psi^3_+)\equiv\Psi, \tag{A.7b}$$

$$\Psi^3=\frac{1}{\sqrt{3}}(\psi^1_++\alpha^*\psi^2_++\alpha\psi^3_+)\equiv\bar{\Psi}, \tag{A.7c}$$

one gets the identity overlap for $\Psi^1(\sigma)$ and the following overlaps for $\Psi(\sigma)$ and $\bar{\Psi}(\sigma)$ [42]

$$-\pi \leq \sigma \leq \pi, \qquad \Psi(\sigma) = \begin{cases} i\alpha^*\Psi(-\sigma-\pi), & -\pi \leq \sigma \leq -\frac{\pi}{2}, \\ -i\alpha\Psi(-\sigma-\pi), & -\frac{\pi}{2} \leq \sigma \leq 0, \\ i\alpha\Psi(\pi-\sigma), & 0 \leq \sigma \leq \frac{\pi}{2}, \\ -i\alpha^*\Psi(\pi-\sigma), & \frac{\pi}{2} \leq \sigma \leq \pi, \end{cases} \tag{A.8}$$

$$-\pi \leq \sigma \leq \pi, \qquad \bar{\Psi}(\sigma) = \begin{cases} i\alpha\bar{\Psi}(-\sigma-\pi), & -\pi \leq \sigma \leq -\frac{\pi}{2}, \\ -i\alpha^*\bar{\Psi}(-\sigma-\pi), & -\frac{\pi}{2} \leq \sigma \leq 0, \\ i\alpha^*\bar{\Psi}(\pi-\sigma), & 0 \leq \sigma \leq \frac{\pi}{2}, \\ -i\alpha\bar{\Psi}(\pi-\sigma), & \frac{\pi}{2} \leq \sigma \leq \pi. \end{cases} \tag{A.9}$$

Overlap equations for $\Psi(\sigma)$ can be rewritten in the component form

$$\Psi_r = -\frac{1}{2}F_{rs}\Psi_s - \frac{1}{2}(\tilde{F}_{rs} - \sqrt{3}C)\Psi_{-s}, \tag{A.10}$$

$$\Psi_{-r} = \frac{1}{2}(\tilde{F}_{rs} - \sqrt{3}C)\Psi_s + \frac{1}{2}F_{rs}\Psi_{-s}. \tag{A.11}$$

We search the three string vertex in the following form

$$|V_3\rangle = \exp\left(\frac{1}{2}\Psi^{1\,\dagger}I\Psi^{1\,\dagger} + \bar{\Psi}^\dagger U\Psi^\dagger\right)|0\rangle_{123}. \tag{A.12}$$

One finds the following equations for $U$

$$U = -\frac{1}{2}FU - \frac{1}{2}(\tilde{F} - \sqrt{3}C), \tag{A.13}$$

$$1 = \frac{1}{2}(\tilde{F} - \sqrt{3}C)U + \frac{1}{2}F. \tag{A.14}$$

They give solutions

$$U = -\frac{\tilde{F} - \sqrt{3}C}{2+F}, \tag{A.15}$$

$$U = \frac{2-F}{\tilde{F} - \sqrt{3}C}. \tag{A.16}$$

These two solutions are equivalent. Solving overlaps for $\bar{\Psi}(\sigma)$ one gets the following equivalent solutions for $CUC$

$$CUC = -\frac{\tilde{F} + \sqrt{3}C}{2+F}, \tag{A.17}$$

$$CUC = \frac{2-F}{\tilde{F} + \sqrt{3}C}. \tag{A.18}$$

# B Conformal Definition of Vertex

Here we present some conformal formulae of [42]. Formula for the state corresponding to the conformal map $\lambda(\xi)$ (4.6) can be rewritten in terms of $x = i\xi$

$$\Lambda_{rs} = \imath^{r+s} \oint \frac{dx}{2\pi\imath} \oint \frac{dx'}{2\pi\imath} x^{-r-\frac{1}{2}} x'^{-s-\frac{1}{2}} \left(\frac{\partial\lambda(x)}{\partial x}\right)^{\frac{1}{2}} \frac{1}{\lambda(x)-\lambda(x')} \left(\frac{\partial\lambda(x')}{\partial x'}\right)^{\frac{1}{2}}. \tag{B.19}$$

For Identity state conformal map has the following form

$$\lambda(x) = \left(i\frac{1+x}{1-x}\right)^2 \equiv z^2. \tag{B.20}$$

Using the generation function defined as

$$\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} = \sum_{n=0}^{\infty} u_n x^n, \quad u_0 = u_1 = 1, \quad u_{2n} = u_{2n+1} = \frac{\left(\frac{2n-1}{2}\right)!}{n!}, \ n \geq 1. \tag{B.21}$$

one can write Identity in the simple form

$$I_{rs} = \imath^{r+s}\left(\frac{I^+_{nm}}{r+s} + \frac{I^-_{nm}}{r-s}\right) = \imath^{r+s}\begin{cases}\left(\frac{-m}{n+m+1} - \frac{m}{n-m}\right) u_n u_m, & n = \text{even},\ m = \text{odd},\\ \left(\frac{n}{n+m+1} - \frac{n}{n-m}\right) u_n u_m, & n = \text{odd},\ m = \text{even},\end{cases} \tag{B.22}$$

where $r = n + \frac{1}{2}$ and $s = m + \frac{1}{2}$.
Fermionic ghosts have Identity equal to $I^{-1}$

$$I^{-1}_{rs} = \imath^{r+s}\left(\frac{I^+_{nm}}{r+s} - \frac{I^-_{nm}}{r-s}\right) = \imath^{r+s}\begin{cases}\left(\frac{-m}{n+m+1} + \frac{m}{n-m}\right) u_n u_m, & n = \text{even},\ m = \text{odd},\\ \left(\frac{n}{n+m+1} + \frac{n}{n-m}\right) u_n u_m, & n = \text{odd},\ m = \text{even}.\end{cases} \tag{B.23}$$

To calculate the fermionic vertex one has the following conformal maps

$$w_a = w_a^0 \left(\frac{1+x}{1-x}\right)^{\frac{2}{3}}, \quad w_1^0 = e^{\imath\frac{\pi}{3}},\ w_2^0 = e^{-\imath\frac{\pi}{3}},\ w_3^0 = e^{-\imath\pi}. \tag{B.24}$$

Using these conformal maps and the generation function defined as

$$\left(\frac{1+x}{1-x}\right)^{\frac{1}{6}} = \sum_{n=0}^{\infty} g_n x^n, \quad g_0 = 1,\ g_1 = \frac{1}{3},\ g_{n+1} = \frac{1}{3(n+1)} g_n + \frac{n-1}{n+1} g_{n-1},\ n \geq 1 \tag{B.25}$$

one gets the following formulae for the three string vertex

$$V^{a\,a}_{rs} = \frac{1}{3} I_{rs} + \imath^{r+s}\left[\frac{M^+_{nm}}{r+s} + \frac{M^-_{nm}}{r-s}\right], \tag{B.26}$$

$$V^{a\,a+1}_{rs} = \frac{1}{2} I_{rs} - \frac{1}{2} V^{aa}_{rs} + \frac{1}{2}\imath^{r+s+1}\left[\frac{\bar{M}^+_{nm}}{r+s} + \frac{\bar{M}^-_{nm}}{r-s}\right], \tag{B.27}$$

$$V^{a\,a-1}_{rs} = \frac{1}{2} I_{rs} - \frac{1}{2} V^{aa}_{rs} - \frac{1}{2}\imath^{r+s+1}\left[\frac{\bar{M}^+_{nm}}{r+s} + \frac{\bar{M}^-_{nm}}{r-s}\right], \tag{B.28}$$

where the matrices $M^{\pm}$ and $\bar{M}^{\pm}$ are defined as follows

$$M^{+}_{nm} = -[(n+1)g_{n+1}(m+1)g_{m+1} - ng_n m g_m][(-)^n - (-)^m], \tag{B.29}$$

$$M^{-}_{nm} = -[ng_n(m+1)g_{m+1} - (n+1)g_{n+1} m g_m][(-)^n - (-)^m], \tag{B.30}$$

$$\bar{M}^{+}_{nm} = [(n+1)g_{n+1}(m+1)g_{m+1} - ng_n m g_m][(-)^n + (-)^m], \tag{B.31}$$

$$\bar{M}^{-}_{nm} = [ng_n(m+1)g_{m+1} - (n+1)g_{n+1} m g_m][(-)^n + (-)^m], \tag{B.32}$$

$$\frac{M^{\pm}_{rs}}{r \pm s}|_{r=s} = 0, \qquad \frac{\bar{M}^{+}_{rs}}{r+s}|_{r=s} = \frac{(n+1)^2 g^2_{n+1} - n^2 g^2_n}{n+\frac{1}{2}}(-)^n, \tag{B.33}$$

$$\frac{\bar{M}^{-}_{rs}}{r-s}|_{r=s} = \frac{2}{3}\sum_{k=0}^{n}(-)^{n-k} g^2_{n-k}. \tag{B.34}$$

This three string vertex can be rewritten in the form (2.10).

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